Optimization of the Matching Network for a Hybrid Coupler Phase Shifter

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Abstract—**A new theory is presented for how to find the optimum data for a transmission line used as a matching network for a hybrid coupler phase shifter. The theory gives a smaller phase-shift error and a greater bandwidth than already known methods. Two methods for equalization of the diode losses for a hybrid coupler phase shifter are described.**

I. Introduction

DIODE phase shifters can be divided into two general groups: the transmission type and the reflection type. The currently used circuits of the transmission type equipped with microwave diodes are: loaded line, switched line, and three-element T or π networks. However, the reflection-type diode phase shifter is used in most applications. The design of the reflection-type phase shifter may be made, for example, around the 3-dB hybrid coupler. The coupler is loaded with identical reflecting diode terminations as shown in Fig. 1. Both types of phase shifters have been described in detail by White [1].

The advantages of the hybrid coupler phase shifter are reciprocal operation with the least number of diodes and the possibility of obtaining any desired phase shift. The transmission match of any hybrid coupler phase shifter is dependent only on the design of the hybrid coupler and, thus, may be performed separately from the design of the matching network.

This paper discusses the use of the transmission line as a matching network for the hybrid coupler phase shifter. It also discusses two methods to equalize the diode losses in the two bias states in a hybrid coupler phase shifter.

II. p–i–n Diode, Equivalent Circuit

In phase-shifter applications, p–i–n diodes are the most extensively used type of microwave diodes. The standard p–i–n diode consists of three layers: heavily doped p^+ and n^+ regions separated by an intrinsic highly resistive I region. The equivalent circuit for a mounted p–i–n diode is presented in [2]. A slightly different circuit is shown in Fig. 2.

In this circuit C_p is the package capacitance, L_p is the package inductance, C_j is the junction capacitance, and R_R and R_F are the residual resistances in the reverse and the forward bias, respectively. L_{ext} is the external inductance which depends on the mount. It may be changed by variation of the diodes' external connections for example, to produce the desired impedances of the diodes.

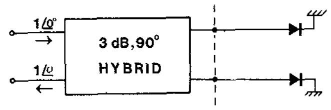

Fig. 1. Hybrid coupler phase shifter with symmetric reflecting diode terminations.

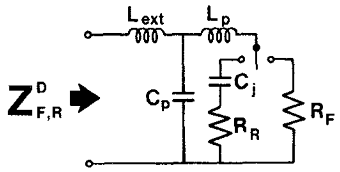

Fig. 2. The equivalent of a mounted p–i–n diode.

The diode impedance in forward bias is

$$Z_F^D = \frac{R_F}{(1-\omega^2 L_p C_p)^2 + \omega^2 C_p^2 R_F^2} + j\left[\omega L_{\text{ext}} + \frac{\omega L_p(1-\omega^2 L_p C_p) - \omega C_p R_F^2}{(1-\omega^2 L_p C_p)^2 + \omega^2 C_p^2 R_F^2}\right]. \tag{1}$$

Usually, for most p–i–n diodes,

$$\omega^2 C_p^2 R_F^2 \ll (1-\omega^2 L_p C_p)^2 \tag{2}$$

and

$$C_p R_F^2 \ll L_p(1-\omega^2 L_p C_p). \tag{3}$$

Therefore,

$$Z_F^D = R_F^D + jX_F^D \cong \frac{R_F}{(1-\omega^2 L_p C_p)^2} + j\left(\omega L_{\text{ext}} + \frac{\omega L_p}{1-\omega^2 L_p C_p}\right). \tag{4}$$

In the reverse bias state,

$$Z_R^D = \frac{R_R}{\left[1-\omega C_p\left(\omega L_p - \frac{1}{\omega C_j}\right)\right]^2 + \omega^2 C_p^2 R_R^2} + j\left(\omega L_{\text{ext}} + \frac{\left(\omega L_p - \frac{1}{\omega C_j}\right)\left[1-\omega C_p\left(\omega L_p - \frac{1}{\omega C_j}\right)\right] - \omega C_p R_R^2}{\left[1-\omega C_p\left(\omega L_p - \frac{1}{\omega C_j}\right)\right]^2 + \omega^2 C_p^2 R_R^2}\right) \tag{5}$$

where

$$\omega^2 R_R^2 C_p^2 \ll \left[1-\omega C_p\left(\omega L_p - \frac{1}{\omega C_j}\right)\right]^2 \tag{6}$$

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$$\omega C_p R_R^2 \ll \left(\omega L_p - \frac{1}{\omega C_j}\right)\left[1 - \omega C_p\left(\omega L_p - \frac{1}{\omega C_j}\right)\right] \quad (7)$$

$$Z_R^D = R_R^D + jX_R^D \cong \frac{R_R}{\left[1 - \omega C_p\left(\omega L_p - \frac{1}{\omega C_j}\right)\right]^2} + j\left(\omega L_{\text{ext}} + \frac{\omega L_p - \frac{1}{\omega C_j}}{1 - \omega C_p\left(\omega L_p - \frac{1}{\omega C_j}\right)}\right). \quad (8)$$

From (4) and (8) the diode components may be calculated:

$$L_{\text{ext}} = \frac{X_F^D}{\omega} - \frac{L_p}{1 - \omega^2 L_p C_p} \quad (9)$$

$$R_F = R_F^D(1 - \omega^2 L_p C_p)^2 \quad (10)$$

$$C_j = \frac{1 + \omega C_p(X_R^D - \omega L_{\text{ext}})}{\omega[(\omega L_{\text{ext}} - X_R^D)(1 - \omega^2 L_p C_p) + \omega L_p]} \quad (11)$$

$$R_R = R_R^D\left[1 - \omega C_p\left(\omega L_p - \frac{1}{\omega C_j}\right)\right]^2. \quad (12)$$

L_p and C_p are specified by the manufacturer of the diode and may be found in data sheets.

III. Matching Network

A matching network consisting of transmission lines and diodes can be designed in many different ways. The single transmission line connecting the diode with the outputs of the hybrid is a very convenient and simple solution, see Fig. 3.

The purpose of the transmission line is to generate the desired phase shift between the two bias states for the diode, i.e., to transform the diode impedances to point A, so that $\angle\Gamma_F - \angle\Gamma_R = \Delta\phi$, the wanted phase shift, is obtained at point A. Point A represents one of the output ports of the hybrid coupler, Fig. 1.

The normalized driving port admittance at point A in the forward bias state is

$$b_F = -\frac{Z_C Z_T - X_F^D h_T}{Z_T X_F^D + Z_T h_T} \quad (13)$$

and in the reverse bias state

$$b_R = -\frac{Z_C Z_T - X_R^D h_T}{Z_T X_R^D + Z_T h_T}. \quad (14)$$

In (13) and (14) the real parts of Z_F^D and Z_R^D are neglected. Z_C is the impedance of the hybrid, and $h_T = \tan\theta_T$. The transmission line is assumed to be lossless. Thus the reflection coefficient Γ is in the forward bias state at point A:

$$\Gamma_F = \frac{1 - jb_F}{1 + jb_F} \quad (15)$$

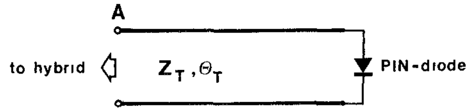

Fig. 3. A transmission line with length θ_T and impedance Z_T as a matching network for a p-i-n diode in a hybrid coupler phase shifter.

and in the reverse bias state at the same point:

$$\Gamma_R = \frac{1 - jb_R}{1 + jb_R}. \quad (16)$$

The phase shift is

$$\Delta\phi = \angle\Gamma_F - \angle\Gamma_R = 2\tan^{-1} b_F - 2\tan^{-1} b_R. \quad (17)$$

After some operations (17) may be rewritten as

$$Z_T^4 h_T^2 \tan\frac{\Delta\phi}{2} + Z_T^3 h_T(X_F^D + X_R^D)\tan\frac{\Delta\phi}{2} - Z_T^2 \cdot\left[Z_C(X_F^D - X_R^D)(1 + h_T^2) - \tan\frac{\Delta\phi}{2}(X_F^2 X_R^2 + Z_C^2)\right] - Z_T Z_C^2 h_T(X_F^D + X_R^D)\tan\frac{\Delta\phi}{2} + Z_C^2 h_T^2 X_F^D X_R^D \tan\frac{\Delta\phi}{2} = 0. \quad (18)$$

Equation (18) may now be used in two different ways: to calculate Z_T at a given θ_T, or to calculate θ_T for some desired Z_T.

The first problem is to solve a fourth-order equation, which may be done by a computer. In a special case, however, for $\theta_T = 90°$, equation (18) simplifies significantly to

$$Z_T^4 - Z_T^2 \frac{(X_F^D - X_R^D)Z_C}{\tan\frac{\Delta\phi}{2}} + Z_C^2 X_F^D X_R^D = 0. \quad (19)$$

Equation (19) is a second-order equation in Z_T^2 with four existing solutions for Z_T. Because Z_T must be real and positive, only one of these four solutions may be used in the design for $\Delta\phi < 180°$:

$$Z_T = \sqrt{\frac{Z_C\left(X_F^D - X_R^D + \sqrt{(X_F^D - X_R^D)^2 - 4X_F^D X_R^D \tan^2\frac{\Delta\phi}{2}}\right)}{2\tan\frac{\Delta\phi}{2}}}. \quad (20)$$

To solve the second problem, i.e., to calculate θ_T for a given Z_T, equation (18) may be rewritten into the following more convenient form:

$$h_T^2\left[Z_T^4 \tan\frac{\Delta\phi}{2} - Z_T^2 Z_C(X_F^D - X_R^D) + Z_C^2 X_F^D X_R^D \tan\frac{\Delta\phi}{2}\right] + h_T Z_T(X_F^D + X_R^D)(Z_T^2 - Z_C^2)\tan\frac{\Delta\phi}{2} - Z_T^2 \cdot\left[Z_C(X_F^D - X_R^D) - \tan\frac{\Delta\phi}{2}(X_F^D X_R^D + Z_C^2)\right] = 0. \quad (21)$$

Equation (21) is a standard second-order equation and it can easily be solved.

IV. The Optimization of Phase Shift Versus Frequency

Equation (18) has an infinite number of solutions for a given diode and desired phase shift. Therefore, it is important to have a criterion on how to choose a transmission line which optimizes the phase shift versus frequency, i.e., the solution to (18) with the minimal phase-shift error and desired phase shift at center frequency. The following set of equations is a solution to this problem:

$$\frac{\Delta\phi}{2} = \tan^{-1} b_F - \tan^{-1} b_R \tag{22a}$$

$$\frac{1}{1+b_F^2}\frac{db_F}{df}\bigg|_{f=f_0} - \frac{1}{1+b_R^2}\frac{db_R}{df}\bigg|_{f=f_0} = 0 \tag{22b}$$

where

$$\frac{db_F}{df}\bigg|_{f=f_0} = \frac{Z_C}{Z_T}\frac{(X_F^{D2}+Z_T^2)\frac{dh_T}{df}\Big|_{f=f_0} + (h_T^2+1)Z_T\frac{dX_F^D}{df}\Big|_{f=f_0}}{(X_F^D+Z_Th_T)^2} \tag{23}$$

and

$$\frac{db_R}{df}\bigg|_{f=f_0} = \frac{Z_C}{Z_T}\frac{(X_R^{D2}+Z_T^2)\frac{dh_T}{df}\Big|_{f=f_0} + (h_T^2+1)Z_T\frac{dX_R^D}{df}\Big|_{f=f_0}}{(X_R^D+Z_Th_T)^2}. \tag{24}$$

(dX_F^D/df) and (dX_R^D/df) may be calculated from the imaginary parts of (4) and (8):

$$\frac{dX_F}{df} = 2\pi L_{\text{ext}} + \frac{2\pi L_p(1+\omega^2L_p^2C_p)}{(1-\omega^2L_pC_p)^2} \tag{25}$$

$$\frac{dX_R}{df} = 2\pi L_{\text{ext}} + 2\pi\frac{\left(L_p+\frac{1}{\omega^2C_j}\right)+C_p\left(\omega L_p-\frac{1}{\omega C_j}\right)^2}{\left[1-\omega C_p\left(\omega L_p-\frac{1}{\omega C_j}\right)\right]^2}. \tag{26}$$

h_T may be expressed as

$$h_T = \tan\left(\theta_{T,f_0}\frac{f}{f_0}\right) \tag{27}$$

and

$$\frac{dh_T}{df} = \frac{1}{\cos^2\theta_{T,f_0}}\frac{\theta_{T,f_0}}{f_0} \tag{28}$$

where θ_{T,f_0} is the electrical length of the transmission line at center frequency.

Inserting (23)–(26) and (28) into (22b) yields:

$$\frac{\left[(X_F^D)^2+Z_T^2\right]\frac{\theta_{T,f_0}}{f_0\cdot\cos^2\theta_{T,f_0}} + Z_T\left[2\pi L_{\text{ext}} + \frac{2\pi L_p(1+\omega^2L_p^2C_p)}{(1-\omega^2L_pC_p)^2}\right]\bigg|_{f=f_0}}{Z_T^2(X_F^D\cos\theta+Z_T\sin\theta)^2+Z_C^2(Z_T\cos\theta-X_F^D\sin\theta)^2}$$
$$= \frac{\left[(X_R^D)^2+Z_T^2\right]\frac{\theta_{T,f_0}}{f_0\cdot\cos^2\theta_{T,f_0}} + Z_T\left[2\pi L_{\text{ext}} + 2\pi\frac{L_p+(1/\omega^2C_j)+C_p(\omega L_p-(1/\omega C_j))^2}{[1-\omega C_p(\omega L_p-(1/\omega C_j))]^2}\right]\bigg|_{f=f_0}}{Z_T^2(X_R^D\cos\theta+Z_T\sin\theta)^2+Z_C^2(Z_T\cos\theta-X_R^D\sin\theta)^2}. \tag{29}$$

Equations (22a) and (29) now constitute the set of equations which has to be solved in order to get the optimum phase shift versus frequency. This solution may be achieved by a computer. Figs. 4 and 5 show the computer-generated phase shift for the 45 and 90° phase-shifter bits. In both cases curve "a" represents a $\theta_{T,f_0} = 90°$ long transformer; curve "b," the optimized one. The phase-shift error depending on the diodes and the matching networks is reduced significantly in both cases.

V. Diode Loss

Because of the resistances presented in each of the diode bias states neither $|\Gamma_F|$ nor $|\Gamma_R|$ are unity. The diode loss comprises a significant part of the total phase shifter loss. It may be as much as about one half of the total loss in the phase shifter.

The loss is generally not the same in the forward and the reverse bias states. It is of great importance in phased array antennas to equalize the phase shifter loss in the two bias states. The equalization results in limiting the amplitude errors of the antenna.

The loss for a hybrid coupler diode phase shifter may be calculated from

$$IL = -20\log|\Gamma|. \tag{30}$$

Γ is the reflection coefficient between the hybrid and the matching network.

The loss balance is achieved when

$$IL_F = IL_R. \tag{31}$$

Equation (31) results in

$$\frac{R_F^L}{R_R^L} = \frac{(R_F^L)^2+(X_F^L)^2+(Z_C)^2}{(R_R^L)^2+(X_R^L)^2+(Z_C)^2} \tag{32}$$

where

$$Z_{F,R}^L = Z_T\frac{Z_{F,R}^D+jZ_T\tan\theta_T}{Z_T+jZ_{F,R}^D\tan\theta_T}. \tag{33}$$

For $\theta_T = 90°$ (32) can be rearranged into

$$\frac{R_F^D}{R_R^D} = \frac{(R_F^D)^2+(X_F^D)^2+\frac{Z_T^4}{Z_C^2}}{(R_R^D)^2+(X_R^D)^2+\frac{Z_T^4}{Z_C^2}}. \tag{34}$$

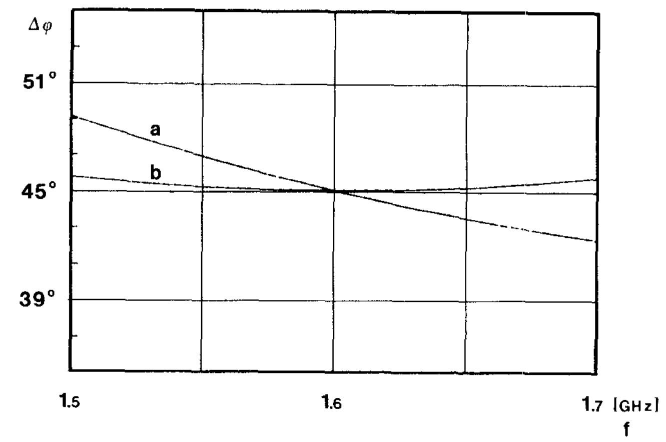

Fig. 4. A computer-generated phase shift for a 45° hybrid coupler phase shifter bit. $X_F^D = j17.5\ \Omega$, $X_R^D = -j50\ \Omega$, $L_p = 1.25$ nH, $C_p = 0.2$ pF, $Z_C = 50\ \Omega$. Curve a: $\theta_{T,f_0} = 90°$, $Z_T = 91.11\ \Omega$. Curve b: $\theta_{T,f_0} = 110.75°$, $Z_T = 91.78\ \Omega$.

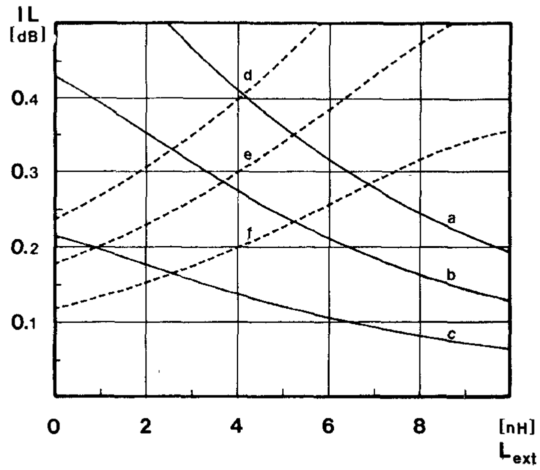

Fig. 6. Diode loss for a 180° hybrid coupler phase shifter bit. $f = 1.6$ GHz, $Z_T = 38.21\ \Omega$, $\theta_{T,f_0} = 90°$, $L_p = 1.25$ nH, $C_p = 0.2$ pF, $X_F^D = j17.5\ \Omega$, $X_R^D = -j50\ \Omega$. Curve a: $R_F = 1.5\ \Omega$. Curve b: $R_F = 1.0\ \Omega$. Curve c: $R_F = 0.5\ \Omega$. Curve d: $R_R = 2\ \Omega$. Curve e: $R_R = 1.5\ \Omega$. Curve f: $R_R = 1.0\ \Omega$.

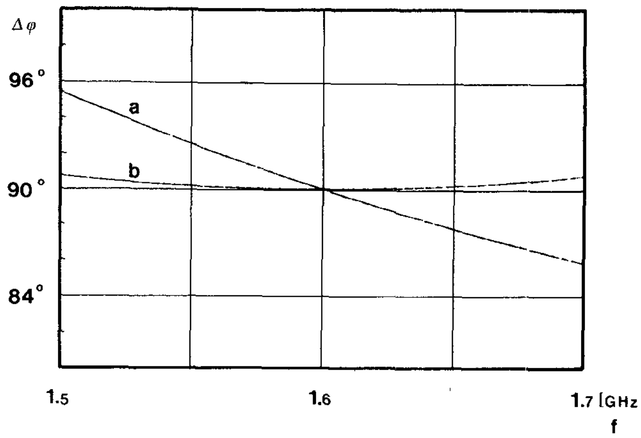

Fig. 5. A computer-generated phase shift for a 90° hybrid coupler phase shifter bit. $X_F^D = j17.5\ \Omega$, $X_R^D = -j50\ \Omega$, $L_p = 1.25$ nH, $C_p = 0.2$ pF, $Z_C = 50\ \Omega$. Curve a: $\theta_{T,f_0} = 90°$, $Z_T = 62.30\ \Omega$. Curve b: $\theta_{T,f_0} = 118.70°$, $Z_T = 62.99\ \Omega$.

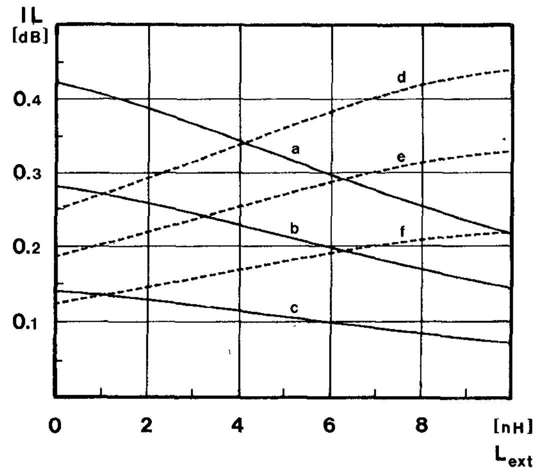

Fig. 7. Diode loss for a 90° hybrid coupler phase shifter bit. $f = 1.6$ GHz, $Z_T = 62.30\ \Omega$, $\theta_{T,f_0} = 90°$, $L_p = 1.25$ nH, $C_p = 0.2$ pF, $X_F^D = j17.5\ \Omega$, $X_R^D = -j50\ \Omega$. Curve a: $R_F = 1.5\ \Omega$. Curve b: $R_F = 1\ \Omega$. Curve c: $R_F = 0.5\ \Omega$. Curve d: $R_R = 2\ \Omega$. Curve e: $R_R = 1.5\ \Omega$. Curve f: $R_R = 1\ \Omega$.

But usually

$$R_F^D \ll \frac{Z_T^2}{Z_C} \tag{35}$$

and

$$R_R^D \ll \frac{Z_T^2}{Z_C} \tag{36}$$

so that

$$\frac{R_F^D}{R_R^D} \cong \frac{(X_F^D)^2 + \dfrac{Z_T^4}{Z_C^2}}{(X_R^D)^2 + \dfrac{Z_T^4}{Z_C^2}}. \tag{37}$$

Equation (32) may now be complemented with (17):

$$\Delta\phi = 2\tan^{-1} b_F - 2\tan^{-1} b_R.$$

The solution to the set of (17) and (32) yields a matching network with the desired phase shift at the center frequency f_0 and an equalized loss in the forward and reverse bias states.

The loss balance for only diodes may be obtained in another way: both X_F^D and X_R^D contain a term depending on L_{ext}. By suitable choice of the diode mounts the external inductance L_{ext} may be chosen so that the insertion loss for the diodes is equal in both bias states for the given Z_T and θ_T. Figs. 6 and 7 show the diode loss versus L_{ext}.

Figs. 6 and 7 show that a proper choice of L_{ext} not only equalizes the diode loss in the forward and reverse bias states but also lowers the loss in the forward bias state. The reduced diode loss may be of the order of 0.1–0.15 dB in a favorable case.

By using (22a,b) and (32) this procedure can be extended to the entire matching network. The matching network obtained in this way gives not only flat shape phase shift versus frequency but even equal insertion loss in the forward and reverse bias states.

VI. Conclusions

The theory presented in this paper results in a new method on how to design the transmission line of a matching network for a hybrid coupler phase shifter. The method uses the diode parasitic elements instead of tuning them out as suggested by Stark [3]. Formulas for optimization of phase shift versus frequency are presented. The theory gives the lower phase-shift error.

This paper also presents the formulas for the balance loss in the forward and reverse bias states. The balance loss gives the diminished loss variation for the phase shifter.

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A Method for Broad-Band Matching of Microstrip Differential Phase Shifters

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Abstract—**Meander-line phase shifters in microstrip are not well matched, because of the phase-velocity difference between the odd and the even mode in the coupled-line region. A stepped impedance design is introduced which allows one to realize, for example, a quarter-wavelength 90° Schiffman phase shifter with three sections, having, theoretically, a return loss better than 30 dB over a 1 : 5 bandwidth. Design equations are given and are confirmed by measurements on microwave integrated circuits in the frequency range 2–10 GHz.**

I. Introduction

One of the most useful passive components of microwave integrated circuits is the differential phase shifter, which is used most frequently for a phase shift of 45, 90, or 180°. Such differential phase shifters are conveniently constructed from parallel-coupled transmission lines connected at one end, as first described by Schiffman [1]. General synthesis procedures were first given by Steenaart [2] and Cristal [3].

The simplest circuit is the type A network [1] or microwave C section, shown in Fig. 1. The meander circuit of Fig. 1 will be perfectly matched at the input, independent of frequency, if the odd and even mode impedances Z_o and Z_e, respectively, conform to

$$Z_o \cdot Z_e = Z_0^2 \qquad (1)$$

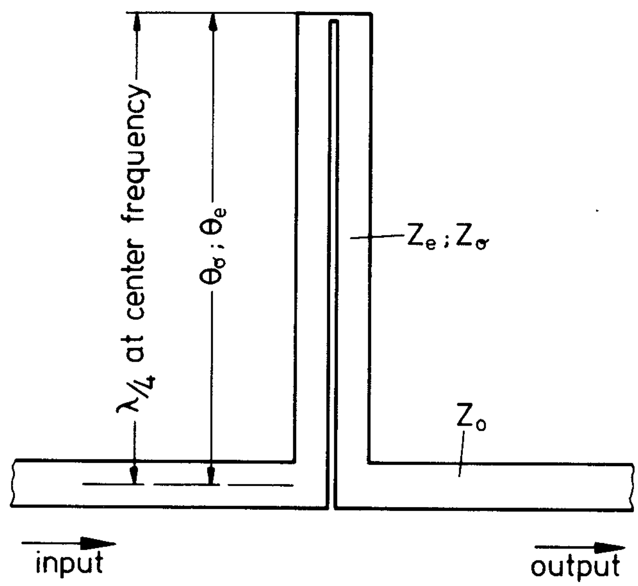

Fig. 1. Microwave C section or meander-line phase shifter visualized in microstrip.

(Z_0 is the characteristic impedance of the connecting transmission line) and if the electrical lengths of the odd and even modes, θ_o and θ_e, respectively, are equal:

$$\theta_o = \theta_e = \theta_i \qquad (2)$$

i.e., ideal TEM behavior.

An example, to which we shall refer repeatedly in this paper, is an octave-wide 90° phase shifter which can be

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